American Journal of Theoretical and Applied Statistics
2022; 11(5): 150-159
http://www.sciencepublishinggroup.com/j/ajtas
doi: 10.11648/j.ajtas.20221105.12
ISSN: 2326-8999 (Print); ISSN: 2326-9006 (Online)

Bayesian and Classical Inferences in Two Inverse Chen Populations Based on Joint Type-II Censoring

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To cite this article:

Shrawan Kumar, Anita Kumari, Kapil Kumar. Bayesian and Classical Inferences in Two Inverse Chen Populations Based on Joint Type-II Censoring. *American Journal of Theoretical and Applied Statistics*. Vol. 11, No. 5, 2022, pp. 150-159. doi: 10.11648/j.ajtas.20221105.12

Received: September 1, 2022; **Accepted**: September 19, 2022; **Published**: September 28, 2022

Abstract: In the era of growing technologies and demand for more reliable products, comparative studies of products from different lines of manufacturing units have become essential. Due to the time-saving and cost-effectiveness properties, the joint Type-II censoring scheme is beneficial for dealing with such types of comparative studies. The inverse Chen distribution has the upside-down or unimodal failure rate function, and it is a suitable lifetime model in life testing and reliability theory. This article contains the Bayesian and classical estimations in the inverse Chen distribution under joint type-II censoring. The maximum likelihood estimators and the corresponding asymptotic confidence intervals of the unknown parameters are developed in the classical estimation approach. In the case of the Bayesian estimation approach, the Bayes estimators of the unknown parameters under the squared error loss function using gamma informative priors are computed. The Bayes estimates are calculated using Markov chain Monte Carlo (MCMC) techniques. Also, the highest posterior density (HPD) credible intervals of the unknown parameters are constructed using MCMC methods. To study various estimates developed in this article, a Monte Carlo simulation study is performed. To compare various estimates, the average estimate, mean squared error values along with the average length and the coverage probabilities are considered. Finally, a real-life problem is analysed to show the applicability of the proposed estimation methods.

Keywords: Inverse Chen Distribution, Joint Type-II Censoring, Maximum Likelihood Estimation, Bayesian Estimation, Monte Carlo Simulation

1. Introduction

The data in the reliability and survival analysis frequently follows a unimodal or upside-down bathtub-shaped failure rate function in real-world scenarios. For such types of problems, researchers developed inverted distributions which possess the upside-down bathtub-shaped failure rate function. Srivastava and Srivastava introduced a two-parameter inverse Chen distribution (ICD) and obtained the maximum likelihood estimate and asymptotic confidence interval of the parameters [22]. Joshi and Pandit estimated the stress strength of the s-out-of-k system for ICD [15]. Agiwal obtained the Bayes estimates of stress-strength reliability for ICD [1]. He obtained different properties of ICD such as failure rate function, r^{th} moment, quantile function, mode, Renyi entropy, and stress-strength reliability. He observed the unimodal failure rate and upside-down bathtub nature of the ICD. He derived the ML and Bayes estimates of the stress-strength reliability under the assumption that the ICD is a robust lifetime distribution.

In recent times, various censoring schemes for different lifetime models have been studied by a number of researchers. For example, Kumar and Garg worked on the estimation of the parameters of generalised inverted Rayleigh distribution under random censoring [20], Chaturvedi and Vyas studied the estimation and testing procedures for Burr distribution under different censoring [7], Chaturvedi et al. discussed the statistical inference in a family of lifetime distributions based on progressively censored data [6], Krishna et al. estimated the stress-strength reliability of inverse Weibull distribution under progressive first failure censoring [17], Kishan and Kumar obtained the

Bayes estimates of the Lindley distribution under progressively censored data with binomial removals [16], Garg et al. studied the estimation of parameters of Lindley distribution under random censoring [10], Garg and Kumar obtained the estimates of $P(Y<X)$ for generalized inverted exponential distribution under hybrid censored data [11], Dhamecha and Patel discussed the estimation of parameters of Kumaraswamy distribution under hybrid censoring [9]. Kumar and Kumar studied the estimation of $P(V<U)$ for inverse Pareto distribution under progressive censoring [19], and many others.

The probability density function (pdf) and cumulative density function (cdf), respectively, of inverse Chen distribution, are given as follows:

$$f(x)=\alpha\theta x^{-(\theta+1)}e^{\left(x^{-\theta}+\alpha\left(1-e^{x^{-\theta}}\right)\right)} \qquad x>0 \tag{1}$$

$$F(x)=e^{\left(\alpha\left(1-e^{x^{-\theta}}\right)\right)} \qquad x>0 \tag{2}$$

The corresponding failure rate function of inverse Chen distribution is given by

$$h(x)=\frac{\alpha\theta x^{-(\theta+1)}e^{\left(x^{-\theta}+\alpha\left(1-e^{x^{-\theta}}\right)\right)}}{1-e^{\left(\alpha\left(1-e^{x^{-\theta}}\right)\right)}} \tag{3}$$

Hereafter, the inverse Chen distribution with parameters $\alpha>0$ and $\theta>0$ is denoted by ICD (α,θ).

As technology is growing in day-to-day life, the products are becoming more reliable, and experimenter gets long lifetime data. For studying such types of lifetime data, life testing, experiments are conducted to get knowledge of the reliability characteristics of different products. To study the reliability characteristics of these highly reliable products the life testing experiments became more time and money-consuming. In literature, numerous censoring schemes are developed to save time and money. In many industries, the products come from different production lines. To test the reliability of production units from different production lines, a comparative study is required. Most of the conventional censoring schemes cover the one-sample problem, while in today's growing world of technologies, comparative studies are much needed.

For comparing the manufactured unit of two different production lines, the joint type-II censoring (JTIIC) scheme was developed by Balakrishnan and Rasouli (2008). The experimenter can save time and money using the JTIIC scheme for the lifetime data of products obtained from two production lines. The real-life examples of using JTIIC scheme are, comparing the lifetimes of two insulated fluids under high voltage. Comparing the lifetimes of different solar panels assembled to provide electricity to the street lights alongside a highway, comparing the air conditioning systems of two boing jet planes and many more. After Balakrishnan and Rasouli [5] many researchers have studied the JTIIC scheme in recent years, see, for example, Ashour and Abo-Kasem studied two generalized exponential populations using Bayesian and non-Bayesian estimation methods [4]. Some recent works on the JTIIC scheme can be found in Al-Matrafi and Abd-Elmougod [2], Asar and Arabi-Belaghi [3], Goel and Krishna [13], and Krishna and Goel [18].

The JTIIC system can be explained such as a life testing experiment where two independent samples of sizes n_1 and n_2 are drawn from two distinct product lines. The lifetimes of the N units are denoted by $X_1,X_2,...,X_{n_1}$ and $Y_1,Y_2,...,Y_{n_2}$ Now, let $Z_1\le Z_2\le...\le Z_N$ denote the order statistic of the random variables $\left(X_1,X_2,...,X_{n_1};Y_1,Y_2,...,Y_{n_2}\right)$, such that $N=n_1+n_2$. Now, mathematically JTIIC is implemented as follows: Let N units be put on a life testing experiment. The failure times of the units $Z_1,Z_2,...,Z_m$ are recorded till the m^{th} failure occur. Here, $m<N$ is a pre-determined number of failures. In JTIIC the data is obtained as (Z,δ) where $Z=(Z_1,Z_2,...,Z_m)$ are the failure times of the units and δ is defined as follows

$$\delta=\begin{cases}1 & Z_i\in X; i=1,2,...,m\\ 0 & otherwise\end{cases}$$

Also, $n_{1m}=\sum_{i=1}^{m}\delta_i$ denotes the number of failures from X and $n_{2m}=\sum_{i=1}^{m}(1-\delta_i)$ denotes the number of failures obtained from Y. The likelihood function of JTIIC is given as follows:

$$L(z,\delta)=c\prod_{i=1}^{m}\{f(z_i)\}^{\delta_i}\{g(z_i)\}^{1-\delta_i}\{\bar{F}(z_m)\}^{n_1-n_{1m}}\{\bar{G}(z_m)\}^{n_2-n_{2m}} \tag{4}$$

To the best of our knowledge, no work has been done on the estimation of parameters of ICD under JTIIC to date. The rest of the article is organized in the following manner: Section 2 contains the Classical estimation with maximum likelihood estimation and asymptotic confidence interval estimation. In section 3, the Bayes estimates of the unknown parameters are calculated under the squared error loss function. MCMC techniques are used to obtain the Bayes estimates of the unknown parameters. A simulation study is carried out in section 4 to compare the different estimates. To illustrate the real-life scenarios, real-life data is analyzed in section 5. Finally, conclusions are made on the basis of the above study in section 6.

2. Classical Estimation

This section contains the classical estimation of the unknown parameters of the ICD under JTIIC. For the classical estimation, the maximum likelihood (ML) estimation method with their corresponding asymptotic confidence interval (ACI) estimates are calculated.

2.1. Maximum Likelihood Estimation

Consider n_1 units of product line A are put through a life testing experiment. The lifetimes of the n_1 units $X_1, X_2, ..., X_{n_1}$ are independent and identically (i.i.d.) distributed random variables from ICD (α_1, θ) with the corresponding pdf $f_X(x)$ and $F_X(x)$ defined in equations (1) and (2) respectively. Similarly, n_2 units of the product line B, are put on a life testing experiment with the lifetimes $Y_1, Y_2, ..., Y_{n_2}$ i.i.d. random variables from ICD (α_2, θ) with the corresponding pdf $g_Y(y)$ and cdf $G_Y(y)$ given in equations (1) and (2), respectively. Now, let $Z_1 \le Z_2 \le ... \le Z_N$ denote the order statistic of the random variables $\left(X_1, X_2, ..., X_{n_1}; Y_1, Y_2, ..., Y_{n_2}\right)$ such that $N = n_1 + n_2$. Now, let (z_i, δ_i); $i = 1, 2, ..., m$ is the joint type-II censored sample obtained from ICD (α_1, θ) and ICD (α_2, θ). Using the pdf and cdf given in equations (1) and (2) the likelihood function given in (4) is obtained as follows:

$$L(z,\delta,\alpha_1,\alpha_2,\theta) = \mathrm{c}\prod_{i=1}^{m}\left\{\alpha_1\theta z_i^{-(\theta+1)} e^{\left(z_i^{-\theta}+\alpha_1(1-e^{z_i^{-\theta}})\right)}\right\}^{\delta_i}\left\{\alpha_2\theta z_i^{-(\theta+1)} e^{\left(z_i^{-\theta}+\alpha_2(1-e^{z_i^{-\theta}})\right)}\right\}^{1-\delta_i}\left\{1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_1-n_{1m}}\left\{1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_2-n_{2m}}$$

$$= c\theta^m \alpha_1^{n_{1m}}\alpha_2^{n_{2m}}\prod_{i=1}^{m} z_i^{-(\theta+1)} e^{\left(z_i^{-\theta}+\left(1-e^{z_i^{-\theta}}(\alpha_1\delta_i+(1-\delta_i)\alpha_2)\right)\right)}\left\{1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_1-n_{1m}}\left\{1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_2-n_{2m}}$$

The log-likelihood function is now obtained by taking the logarithm of the aforementioned equation and taking the following form:

$$\begin{aligned}\ln \mathrm{L} = {} & \ln(\mathrm{c}) + \mathrm{m}\ln(\theta) + \mathrm{n}_{1m}\ln(\alpha_1) + \mathrm{n}_{2m}\ln(\alpha_2) - (\theta+1)\sum_{i=1}^{m}\ln\left(z_i\right) + \\ & \sum_{i=1}^{m} z_i^{-\theta} + \sum_{i=1}^{m}\left(1-e^{z_i^{-\theta}}\left(\alpha_1\delta_i+(1-\delta_i)\alpha_2\right)\right) + \left(\mathrm{n}_1-\mathrm{n}_{1m}\right)\ln\left(1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right) + \left(\mathrm{n}_2-\mathrm{n}_{2m}\right)\ln\left(1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right)\end{aligned} \tag{5}$$

To obtain the ML estimates of α_1, α_2 and θ we differentiate equation (5) with respect to α_1, α_2 and θ respectively and put them equal to zero. The three normal equations are obtained as follows:

$$\frac{\partial \ln L}{\partial \alpha_1} = \frac{n_{1m}}{\alpha_1} + \sum_{i=1}^{m}\delta_i\left(1-e^{z_i^{-\theta}}\right) - \frac{\left(n_1-n_{1m}\right)\left(1-e^{z_m^{-\theta}}\right)e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}}{\left(1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right)} = 0$$

$$\frac{\partial \ln L}{\partial \alpha_2} = \frac{n_{2m}}{\alpha_2} + \sum_{i=1}^{m}\left(1-\delta_i\right)\left(1-e^{z_i^{-\theta}}\right) - \frac{\left(n_2-n_{2m}\right)\left(1-e^{z_m^{-\theta}}\right)e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}}{\left(1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right)} = 0$$

$$\begin{aligned}\frac{\partial \ln L}{\partial \theta} = {} & \frac{m}{\theta} - \sum_{i=1}^{m}\ln(z_i) - \sum_{i=1}^{m} z_i^{-\theta}\ln(z_i) + \sum_{i=1}^{m}\left[\alpha_1\delta_i + (1-\delta_i)\alpha_2\right]e^{z_i^{-\theta}} z_i^{-\theta}\ln(z_i) \\ & -\left(\mathrm{n}_1-\mathrm{n}_{1m}\right)\frac{\alpha_1 z_m^{-\theta}\ln(z_m)e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}e^{z_m^{-\theta}}}{\left(1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right)} - \left(\mathrm{n}_2-\mathrm{n}_{2m}\right)\frac{\alpha_2 z_m^{-\theta}\ln(z_m)e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}e^{z_m^{-\theta}}}{\left(1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right)} = 0\end{aligned}$$

From the above normal equations it is clear that we can not get the explicit solutions for the ML estimates, $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\theta}$ of α_1, α_2 and θ respectively. To obtain the ML estimates we use some iterative methods such as the Newton-Raphson method etc.

2.2. Asymptotic Confidence Interval

Here, in this subsection, we obtain the asymptotic confidence intervals of α_1, α_2 and θ using the observed Fisher information matrix. The observed Fisher information matrix under JTIIC is given as follows:

$$I(\hat{\alpha}_1,\hat{\alpha}_2,\hat{\theta}) = \begin{bmatrix} -\frac{\partial^2 \ln L}{\partial \alpha_1^2} & -\frac{\partial^2 \ln L}{\partial \alpha_1 \partial \alpha_2} & -\frac{\partial^2 \ln L}{\partial \alpha_1 \partial \theta} \\ -\frac{\partial^2 \ln L}{\partial \alpha_2 \partial \alpha_1} & -\frac{\partial^2 \ln L}{\partial \alpha_2^2} & -\frac{\partial^2 \ln L}{\partial \alpha_2 \partial \theta} \\ -\frac{\partial^2 \ln L}{\partial \theta \partial \alpha_1} & -\frac{\partial^2 \ln L}{\partial \theta \partial \alpha_2} & -\frac{\partial^2 \ln L}{\partial \theta^2} \end{bmatrix}_{\alpha_1=\hat{\alpha}_1,\alpha_2=\hat{\alpha}_2,\theta=\hat{\theta}} = -\begin{bmatrix} \hat{I}_{11} & \hat{I}_{12} & \hat{I}_{13} \\ \hat{I}_{21} & \hat{I}_{22} & \hat{I}_{23} \\ \hat{I}_{31} & \hat{I}_{32} & \hat{I}_{33} \end{bmatrix} \tag{6}$$

Here, $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\theta}$ are the ML estimates of α_1, α_2 and θ respectively. The elements of the observed Fisher information matrix are given by

$$I_{11} = -\frac{n_{1m}}{\alpha_1^2} - \frac{(n_1-n_{1m})\left(1-e^{z_m^{-\theta}}\right)^2 e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}}{\left(1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right)^2}, \quad I_{22} = -\frac{n_{2m}}{\alpha_2^2} - \frac{(n_2-n_{2m})\left(1-e^{z_m^{-\theta}}\right)^2 e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}}{\left(1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right)^2},$$

$$I_{13} = \sum_{i=1}^{m} \delta_i e^{z_i^{-\theta}} z_i^{-\theta} \ln(z_i) - (\mathrm{n}_1-\mathrm{n}_{1m}) \frac{e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)} e^{z_m^{-\theta}} z_m^{-\theta} \ln(z_m)\left(1+\alpha_1\left(1-e^{z_m^{-\theta}}\right)-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right)}{\left(1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right)^2} = I_{31}$$

$$I_{23} = \sum_{i=1}^{m} (1-\delta_i) e^{z_i^{-\theta}} z_i^{-\theta} \ln(z_i) - (\mathrm{n}_2-\mathrm{n}_{2m}) \frac{e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)} e^{z_m^{-\theta}} z_m^{-\theta} \ln(z_m)\left(1+\alpha_2\left(1-e^{z_m^{-\theta}}\right)-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right)}{\left(1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right)^2} = I_{32}$$

$$I_{12} = I_{21} = 0$$

$$\begin{aligned} I_{33} = &-\frac{m}{\theta^2} - \sum_{i=1}^{m} z_i^{-\theta} \ln^2(z_i) - \sum_{i=1}^{m} \left[\alpha_1\delta_i + (1-\delta_i)\alpha_2\right] e^{z_i^{-\theta}} z_i^{-\theta} \ln^2(z_i)\left(1+z_i^{-\theta}\right) \\ &- \frac{\alpha_1(\mathrm{n}_1-\mathrm{n}_{1m}) z_m^{-\theta} \ln^2(z_m) e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)} e^{z_m^{-\theta}} \left(\alpha_1 e^{z_m^{-\theta}} z_m^{-\theta} - z_m^{-\theta} - 1 + e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}(1+z_m^{-\theta})\right)}{\left(1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right)^2} \\ &- \frac{\alpha_2(\mathrm{n}_2-\mathrm{n}_{2m}) z_m^{-\theta} \ln^2(z_m) e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)} e^{z_m^{-\theta}} \left(\alpha_2 e^{z_m^{-\theta}} z_m^{-\theta} - z_m^{-\theta} - 1 + e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}(1+z_m^{-\theta})\right)}{\left(1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right)^2} \end{aligned}$$

The estimated variances of $\hat{\alpha}_1$, $\hat{\alpha}_2$ and $\hat{\theta}$ are given as

$$\hat{V}ar(\hat{\alpha}_1) = -\frac{1}{\hat{I}_{11}} \ \& \ \hat{V}ar(\hat{\alpha}_2) = -\frac{1}{\hat{I}_{22}} \ \& \ \hat{V}ar(\hat{\theta}) = -\frac{1}{\hat{I}_{33}} \tag{7}$$

The $100(1-\gamma)\%$ asymptotic confidence intervals for $(\alpha_1, \alpha_2, \theta)$ using the approximated standard normal distribution are given as

$$P\left[\left|\frac{(\alpha_1 - \hat{\alpha}_1)}{\sqrt{\hat{V}ar(\hat{\alpha}_1)}}\right| \le Z_{\frac{\gamma}{2}}\right] \ \& \ P\left[\left|\frac{(\alpha_2 - \hat{\alpha}_2)}{\sqrt{\hat{V}ar(\hat{\alpha}_2)}}\right| \le Z_{\frac{\gamma}{2}}\right] \ \& \ P\left[\left|\frac{(\theta - \hat{\theta})}{\sqrt{\hat{V}ar(\hat{\theta})}}\right| \le Z_{\frac{\gamma}{2}}\right] \tag{8}$$

Here, $Z_{\frac{\gamma}{2}}$ is the upper $\frac{\gamma}{2}$ percentile of standard normal distribution.

3. Bayesian Estimation

This section is devoted to calculating the Bayes estimates of α_1, α_2 and θ. The Bayes estimates are obtained using the squared error loss function (SELF). The Bayes estimate under SELF is the mean of the posterior distribution. To calculate the Bayes estimates of α_1, α_2 and θ let us assume that α_1, α_2 and θ have gamma priors with hyperparameters (a_1, b_1), (a_2, b_2) and (a_3, b_3) respectively. The prior densities of the unknown parameters are given by:

$$\Pi(\alpha_1) = \frac{b_1^{a_1}}{\Gamma(a_1)} \alpha_1^{a_1 - 1} e^{-b_1 \alpha_1} \ ; \alpha_1 > 0, a_1, b_1 > 0, \ \Pi(\alpha_2) = \frac{b_2^{a_2}}{\Gamma(a_2)} \alpha_2^{a_2 - 1} e^{-b_2 \alpha_2} \ ; \alpha_2 > 0, a_2, b_2 > 0,$$

and

$$\Pi(\theta) = \frac{b_3^{a_3}}{\Gamma(a_3)} \theta^{a_3 - 1} e^{-b_3 \theta} \ ; \theta > 0, a_3, b_3 > 0$$

The joint prior distribution of α_1, α_2 and θ is given by:

$$\Pi(\alpha_1, \alpha_2, \theta) \propto \alpha_1^{a_1 - 1} e^{-b_1 \alpha_1} \ \alpha_2^{a_2 - 1} e^{-b_2 \alpha_2} \ \theta^{a_3 - 1} e^{-b_3 \theta} \ ; \alpha_1, \alpha_2, \theta > 0, a_i, b_i > 0; i = 1, 2, 3$$

Now, the joint posterior distribution of α_1, α_2 and θ is given by:

$$\Pi(\alpha_1, \alpha_2, \theta \mid data) = \frac{L(data \mid \alpha_1, \alpha_2, \theta) \Pi(\alpha_1, \alpha_2, \theta)}{\int_0^\infty \int_0^\infty \int_0^\infty L(data \mid \alpha_1, \alpha_2, \theta) \Pi(\alpha_1, \alpha_2, \theta) d\alpha_1 d\alpha_2 \theta}$$

$$= K^{-1} c \theta^{m + a_3 - 1} \alpha_1^{n_{1m} + a_1 - 1} \alpha_2^{n_{2m} + a_2 - 1} e^{-(b_1 \alpha_1 + b_2 \alpha_2 + b_3 \theta)} \prod_{i=1}^{m} z_i^{-(\theta + 1)} e^{\left(z_i^{-\theta} + \left(1 - e^{z_i^{-\theta}} (\alpha_1 \delta_i + (1 - \delta_i) \alpha_2)\right)\right)}$$

$$\left\{1 - e^{\alpha_1 \left(1 - e^{z_m^{-\theta}}\right)}\right\}^{n_1 - n_{1m}} \left\{1 - e^{\alpha_2 \left(1 - e^{z_m^{-\theta}}\right)}\right\}^{n_2 - n_{2m}} \tag{9}$$

where,

$$K^{-1} = \iiint \theta^{m + a_3 - 1} \alpha_1^{n_{1m} + a_1 - 1} \alpha_2^{n_{2m} + a_2 - 1} e^{-(b_1 \alpha_1 + b_2 \alpha_2 + b_3 \theta)} \prod_{i=1}^{m} z_i^{-(\theta + 1)} e^{\left(z_i^{-\theta} + \left(1 - e^{z_i^{-\theta}} (\alpha_1 \delta_i + (1 - \delta_i) \alpha_2)\right)\right)} \left\{1 - e^{\alpha_1 \left(1 - e^{z_m^{-\theta}}\right)}\right\}^{n_1 - n_{1m}} \left\{1 - e^{\alpha_2 \left(1 - e^{z_m^{-\theta}}\right)}\right\}^{n_2 - n_{2m}} d\alpha_1 d\alpha_2 \theta$$

From equation (9) the closed form solution for the Bayes estimates $\tilde{\alpha}_1$, $\tilde{\alpha}_2$ and $\tilde{\theta}$ of α_1, α_2 and θ cannot be calculated. Therefore, to calculate the Bayes estimate of unknown parameters, the MCMC techniques are used.

3.1. MCMC Techniques

This subsection uses MCMC techniques to generate random numbers from the marginal posterior distribution functions. The Metropolis-Hastings algorithm is used to generate the random numbers from the marginal posterior distributions of α_1, α_2 and θ. The marginal posterior distributions of α_1, α_2 and θ are given as follows

$$\Pi_1(\alpha_1 \mid \theta, data) = \alpha_1^{n_{1m}+a_1-1} \prod_{i=1}^{m} e^{\alpha_1\left(\delta_i\left(1-e^{z_i^{-\theta}}\right)-b_1\right)} \left\{1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_1-n_{1m}} \tag{10}$$

$$\Pi_2(\alpha_2 \mid \theta, data) = \alpha_2^{n_{2m}+a_2-1} \prod_{i=1}^{m} e^{\alpha_2\left(\delta_i\left(1-e^{z_i^{-\theta}}\right)-b_2\right)} \left\{1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_2-n_{2m}} \tag{11}$$

$$\Pi_3(\theta \mid \alpha_1, \alpha_2, data) = \theta^{m+a_3-1} \prod_{i=1}^{m} z_i^{-(\theta+1)} e^{\left(z_i^{-\theta}+\left(1-e^{z_i^{-\theta}}\left(\alpha_1\delta_i+(1-\delta_i)\alpha_2\right)\right)\right)} \left\{1-e^{\alpha_1\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_1-n_{1m}} \left\{1-e^{\alpha_2\left(1-e^{z_m^{-\theta}}\right)}\right\}^{n_2-n_{2m}} \tag{12}$$

From equations (10), (11) and (12) it is clear that the marginal distributions of α_1, α_2 and θ respectively do not follow a well known distribution. The MCMC techniques are used to generate the random sample from (10), (11) and (12). A detailed discussion about MCMC and M-H algorithm can be found in Metropolis and Ulam [21], Hastings [14] and Gelman et al. [12]. The steps for the Gibbs sampling are given below:

Step 1. Start with initial guess $\alpha_1^{(0)}$, $\alpha_2^{(0)}$, $\theta^{(0)}$.

Step 2. Generate $\alpha_1^{(j)}$ from $\Pi_1(\alpha_1 \mid \theta^{(j)}, data)$ given in (10) using M-H algorithm with proposal density as normal density.

Step 3. Generate $\alpha_2^{(j)}$ from $\Pi_2(\alpha_2 \mid \theta^{(j)}, data)$ given in (11) using M-H algorithm with proposal density as normal density.

Step 4. Generate $\theta^{(j)}$ from $\Pi_3(\theta \mid \alpha_1^{(j)}, \alpha_2^{(j)}, data)$ given in (12) using M-H algorithm with proposal density as normal density.

Step 5. For $j = 1, 2, ..., M$ Repeat steps 2-5, to get the sequence of the parameter α_1, α_2 and θ as $(\alpha_{11}, \alpha_{12}, ..., \alpha_{1M})$, $(\alpha_{21}, \alpha_{22}, ..., \alpha_{2M})$, $(\theta_1, \theta_2, ..., \theta_M)$.

First M_0, α_{1j}'s; $j = 1, 2, ..., M_0$ are discarded to generate a random sample from the stationary Markov chain distribution, which is often the posterior distribution, where M_0 is known as the burn-in period. The following equations provide the Bayes estimates for the parameters α_1, α_2 and θ under SELF are given by

$$\hat{\alpha}_{1MH} = \frac{1}{M-M_0} \sum_{j=M_0+1}^{M} \alpha_{1j} \tag{13}$$

$$\hat{\alpha}_{2MH} = \frac{1}{M-M_0} \sum_{j=M_0+1}^{M} \alpha_{2j} \tag{14}$$

$$\hat{\theta}_{MH} = \frac{1}{M-M_0} \sum_{j=M_0+1}^{M} \theta_j \tag{15}$$

3.2. HPD Credible Intervals

Here, the HPD credible interval for α_1, α_2 and θ are constructed using the algorithm proposed by Chen and Shao [8]. Let $\alpha_{1(1)}, \alpha_{1(2)}, ..., \alpha_{1(M-M_0)}$ denote the ordered values of $\alpha_{1(M_0+1)}, \alpha_{1(M_0+2)}, ..., \alpha_{1(M)}$, then the $100(1-\xi)\%$ HPD credible interval for α_1 is obtained as $(\alpha_{1(j)}, \alpha_{1(j+[(1-\xi)\mathrm{M}])})$, here, j is chosen such that

$$(\alpha_{1(j+[(1-\xi)(\mathrm{M}-\mathrm{M}_0)])} - \alpha_{1j}) = \min_{1 \le j \le \gamma M} (\alpha_{1(j+[(1-\xi)(\mathrm{M}-\mathrm{M}_0)])} - \alpha_{1j}),$$

$j = 1, 2, ..., (M-M_0)$, where $[x]$ is the integer part of x. Similarly, the $100(1-\xi)\%$ HPD credible interval for α_2 and θ is obtained as $(\alpha_{2(j)}, \alpha_{2(j+[(1-\xi)\mathrm{M}])})$, $(\theta_{(j)}, \theta_{(j+[(1-\xi)\mathrm{M}])})$, where j is chosen such that

$$(\alpha_{2(j+[(1-\xi)(\mathrm{M}-\mathrm{M}_0)])} - \alpha_{2j}) = \min_{1 \le j \le \gamma M} (\alpha_{2(j+[(1-\xi)(\mathrm{M}-\mathrm{M}_0)])} - \alpha_{2j}), \quad j = 1, 2, ..., (M-M_0).$$

$$(\theta_{(j+[(1-\xi)(\mathrm{M}-\mathrm{M}_0)])} - \theta_j) = \min_{1 \le j \le \gamma M} (\theta_{(j+[(1-\xi)(\mathrm{M}-\mathrm{M}_0)])} - \theta_j), \quad j = 1, 2, ..., (M-M_0).$$

4. Simulation Study

A simulation study to compare different estimates of the unknown parameters is carried out in this section. The average estimate (AE) and mean squared error (MSE) of the ML and Bayes estimates of the unknown parameters from ICD (α_1, θ)

and ICD (α_2,θ) under joint type-II censoring are calculated. For computing, the Bayes estimates informative priors are used. For the interval estimates, the average length (AL) and coverage probability (CP) of ACI and HPD credible intervals are calculated. For the simulation study, two sets of true values of the parameters are chosen, such as $(\alpha_1,\alpha_2,\theta)=(0.5,0.75,1)$ and $(0.75,1.5,1)$ respectively. The values of hyperparameters are chosen in such a way that the mean of the prior density becomes equal to the true value of the parameter. For both the set of true values the values of hyperparameters are given by $a_1=2$, $b_1=4$, $a_2=3$, $b_2=4$, $a_3=4$, $b_3=4$ and $a_1=3$, $b_1=4$, $a_2=3$, $b_2=2$, $a_3=4$, $b_3=4$ respectively. To calculate the Bayes estimates using MCMC techniques $10,000$ MCMC samples are generated and $2,000$ are taken as a burn-in period. Different estimates obtained under different values of n_1,n_2 and m are shown in Tables 1, 2, 3 and 4.

From the simulation results obtained in Tables 1, 2, 3, and 4. We can conclude that both ML and Bayes estimates provide good estimates of the unknown parameters of ICD in the case of joint type-II censored data with the MSEs less than 0.10 in all the cases. From Tables 1 and 3 we can say that when the sample size is increased, the MSEs of both ML and Bayes estimates decrease. The Bayes estimates perform better than ML estimates in view of MSEs. The obtained Bayes estimates provide lesser MSEs than the ML estimates. From Tables 2 and 4 we can say that the AL of both ACI and HPD credible intervals decreases when we increase the sample size. The HPD credible intervals provide lesser AL than the ACI. All the intervals attain their nominal level of significance.

Table 1. *The AEs and MSEs of the ML and Bayes estimates of* α_1,α_2 *and* θ *when* $(\alpha_1,\alpha_2,\theta)=(0.5,0.75,1)$.

(n_1,n_2,m)	$\hat{\alpha}_1$				$\hat{\alpha}_2$				$\hat{\theta}$			
	MLE		**Bayes**		**MLE**		**Bayes**		**MLE**		**Bayes**	
	AE	**MSE**	**AE**	**MSE**	**AE**	**MSE**	**AE**	**MSE**	**AE**	**MSE**	**AE**	**MSE**
(10,15,18)	0.5249	0.0450	0.5318	0.0220	0.8117	0.0716	0.7943	0.0322	1.1200	0.0626	1.0573	0.0256
(10,15,20)	0.5460	0.0646	0.5404	0.0240	0.8061	0.0690	0.7888	0.0322	1.1143	0.0627	1.0577	0.0276
(15,20,26)	0.5128	0.0264	0.5236	0.0160	0.7750	0.0427	0.7731	0.0244	1.0860	0.0344	1.0458	0.0173
(15,20,30)	0.5249	0.0260	0.5326	0.0160	0.7759	0.0440	0.7714	0.0255	1.0786	0.0318	1.0447	0.0169
(20,25,34)	0.5099	0.0181	0.5206	0.0121	0.7760	0.0303	0.7753	0.0200	1.0610	0.0217	1.0322	0.0114
(20,25,38)	0.5132	0.0179	0.5220	0.0124	0.7722	0.0293	0.7710	0.0196	1.0598	0.0222	1.0344	0.0128
(25,30,42)	0.5082	0.0130	0.5172	0.0093	0.7695	0.0241	0.7698	0.0168	1.0485	0.0170	1.0267	0.0096
(25,30,48)	0.5072	0.0141	0.5141	0.0104	0.7644	0.0218	0.7631	0.0159	1.0415	0.0146	1.0232	0.0089
(30,35,52)	0.5080	0.0110	0.5148	0.0081	0.7681	0.0208	0.7665	0.0156	1.0354	0.0131	1.0188	0.0079
(30,35,60)	0.5067	0.0113	0.5134	0.0083	0.7658	0.0200	0.7649	0.0152	1.0365	0.0121	1.0211	0.0075
(40,45,68)	0.5040	0.0085	0.5108	0.0067	0.7638	0.0155	0.7646	0.0123	1.0322	0.0102	1.0187	0.0063
(40,45,75)	0.5057	0.0085	0.5117	0.0067	0.7640	0.0149	0.7645	0.0118	1.0298	0.0094	1.0178	0.0060
(50,55,80)	0.5058	0.0062	0.5117	0.0051	0.7602	0.0120	0.7619	0.0101	1.0261	0.0073	1.0151	0.0046
(50,55,85)	0.5033	0.0062	0.5094	0.0050	0.7608	0.0114	0.7626	0.0095	1.0260	0.0074	1.0151	0.0048

Table 2. *The AL and CP of ACI and HPD credible intervals of* α_1,α_2 *and* θ *when* $(\alpha_1,\alpha_2,\theta)=(0.5,0.75,1)$.

(n_1,n_2,m)	$\hat{\alpha}_1$				$\hat{\alpha}_2$				$\hat{\theta}$			
	ACI		**HPD**		**ACI**		**HPD**		**ACI**		**HPD**	
	AL	**CP**	**AL**	**CP**	**AL**	**CP**	**AL**	**CP**	**AL**	**CP**	**AL**	**CP**
(10,15,18)	0.7008	0.920	0.5816	0.963	0.8479	0.937	0.7161	0.976	0.7456	0.931	0.6610	0.985
(10,15,20)	0.7234	0.916	0.5903	0.964	0.8402	0.940	0.7101	0.974	0.7218	0.916	0.6437	0.972
(15,20,26)	0.5598	0.921	0.4849	0.952	0.6976	0.932	0.6188	0.963	0.5985	0.934	0.5492	0.974
(15,20,30)	0.5663	0.930	0.4923	0.968	0.6957	0.931	0.6161	0.955	0.5724	0.925	0.5284	0.976
(20,25,34)	0.4822	0.922	0.4245	0.958	0.6215	0.934	0.5629	0.964	0.5109	0.947	0.4783	0.982
(20,25,38)	0.4821	0.932	0.4252	0.955	0.6169	0.945	0.5594	0.958	0.4944	0.940	0.4638	0.974
(25,30,42)	0.4298	0.938	0.3807	0.960	0.5618	0.942	0.5151	0.956	0.4526	0.941	0.4283	0.977
(25,30,48)	0.4265	0.939	0.3789	0.947	0.5561	0.944	0.5103	0.950	0.4326	0.949	0.4106	0.977
(30,35,52)	0.3916	0.943	0.3487	0.951	0.5178	0.952	0.4776	0.952	0.4059	0.942	0.3875	0.975
(30,35,60)	0.3880	0.932	0.3476	0.957	0.5151	0.941	0.4765	0.954	0.3910	0.936	0.3741	0.973
(40,45,68)	0.3367	0.933	0.3023	0.947	0.4537	0.937	0.4246	0.952	0.3513	0.929	0.3370	0.970
(40,45,75)	0.3364	0.943	0.3029	0.949	0.4525	0.942	0.4242	0.958	0.3423	0.932	0.3295	0.965
(50,55,80)	0.3031	0.944	0.2724	0.946	0.4086	0.949	0.3847	0.958	0.3181	0.950	0.3070	0.979
(50,55,85)	0.3011	0.941	0.2710	0.957	0.4079	0.945	0.3855	0.953	0.3127	0.933	0.3068	0.973

Table 3. *The AEs and MSEs of the ML and Bayes estimates of* α_1, α_2 *and* θ *when* $(\alpha_1, \alpha_2, \theta) = (0.75, 1.5, 1)$.

(n_1, n_2, m)	$\hat{\alpha}_1$				$\hat{\alpha}_2$				$\hat{\theta}$			
	MLE		Bayes		MLE		Bayes		MLE		Bayes	
	AE	MSE	AE	MSE	AE	MSE	AE	MSE	AE	MSE	AE	MSE
(10,15,18)	0.8291	0.117	0.7896	0.0398	1.7915	0.5450	1.5886	0.1295	1.1413	0.0865	1.0635	0.0302
(10,15,20)	0.8652	0.2434	0.8008	0.0426	1.7768	0.5409	1.5775	0.1287	1.1345	0.0871	1.0612	0.0303
(15,20,26)	0.7983	0.0671	0.7809	0.0312	1.6593	0.2703	1.5464	0.0979	1.1006	0.0462	1.0560	0.0217
(15,20,30)	0.8131	0.0620	0.7940	0.0308	1.6497	0.2457	1.5432	0.1018	1.0892	0.0407	1.0520	0.0207
(20,25,34)	0.7832	0.0397	0.7780	0.0244	1.6270	0.1659	1.5507	0.0807	1.0716	0.0290	1.0427	0.0157
(20,25,38)	0.7894	0.0418	0.7801	0.0250	1.6180	0.1571	1.5422	0.0786	1.0682	0.0287	1.0420	0.0162
(25,30,42)	0.7775	0.0285	0.7738	0.0191	1.5987	0.1303	1.5403	0.0672	1.0562	0.0225	1.0352	0.0135
(25,30,48)	0.7743	0.0325	0.7693	0.0216	1.5786	0.1103	1.5263	0.0635	1.0468	0.0184	1.0289	0.0114
(30,35,52)	0.7726	0.0233	0.7707	0.0169	1.5809	0.1139	1.5334	0.0624	1.0406	0.0165	1.0245	0.0103
(30,35,60)	0.7711	0.0243	0.7689	0.0174	1.5765	0.1018	1.5293	0.0608	1.0412	0.0150	1.0269	0.0099
(40,45,68)	0.7728	0.0166	0.7679	0.0168	1.5815	0.0747	1.5338	0.0571	1.0367	0.0122	1.0271	0.0096
(40,45,75)	0.7773	0.0178	0.7648	0.0138	1.5604	0.0689	1.5164	0.0434	1.0277	0.0111	1.0209	0.0082
(50,55,80)	0.7671	0.0137	0.7640	0.0113	1.5459	0.0603	1.5251	0.0401	1.0272	0.0094	1.0187	0.0064
(50,55,85)	0.7594	0.0133	0.7619	0.0114	1.5553	0.0609	1.5197	0.0394	1.0279	0.0091	1.0263	0.0065

Table 4. *The AL and CP of ACI and HPD credible intervals of* α_1, α_2 *and* θ *when* $(\alpha_1, \alpha_2, \theta) = (0.75, 1.5, 1)$.

(n_1, n_2, m)	$\hat{\alpha}_1$				$\hat{\alpha}_2$				$\hat{\theta}$			
	ACI		HPD		ACI		HPD		ACI		HPD	
	AL	CP	AL	CP	AL	CP	AL	CP	AL	CP	AL	CP
(10,15,18)	1.0638	0.942	0.8321	0.970	2.0546	0.977	1.4350	0.977	0.8642	0.944	0.7366	0.983
(10,15,20)	1.1198	0.945	0.8440	0.973	2.0287	0.974	1.4246	0.974	0.8272	0.926	0.7099	0.980
(15,20,26)	0.8300	0.929	0.7031	0.963	1.5884	0.964	1.2405	0.961	0.6874	0.942	0.6199	0.981
(15,20,30)	0.8398	0.946	0.7155	0.977	1.5693	0.956	1.2353	0.958	0.6458	0.939	0.5885	0.974
(20,25,34)	0.7011	0.940	0.6212	0.966	1.3737	0.965	1.1284	0.968	0.5819	0.946	0.5397	0.978
(20,25,38)	0.7048	0.937	0.6234	0.959	1.3603	0.956	1.1218	0.960	0.5574	0.948	0.5193	0.974
(25,30,42)	0.6209	0.950	0.5609	0.966	1.2208	0.953	1.0346	0.957	0.5139	0.947	0.4837	0.972
(25,30,48)	0.6182	0.941	0.5585	0.956	1.1994	0.959	1.0232	0.955	0.484	0.951	0.4578	0.974
(30,35,52)	0.5624	0.946	0.5153	0.954	1.1138	0.949	0.9591	0.953	0.4568	0.946	0.4345	0.976
(30,35,60)	0.5604	0.940	0.5142	0.961	1.1048	0.948	0.9561	0.958	0.4348	0.941	0.4155	0.968
(40,45,68)	0.4862	0.964	0.4662	0.936	0.9774	0.961	0.8823	0.947	0.3971	0.945	0.3894	0.956
(40,45,75)	0.4884	0.950	0.4479	0.959	0.9599	0.953	0.8436	0.959	0.3788	0.946	0.3662	0.966
(50,55,80)	0.4316	0.943	0.4038	0.955	0.8598	0.938	0.7722	0.950	0.3510	0.940	0.3395	0.973
(50,55,85)	0.4277	0.950	0.4018	0.955	0.8652	0.953	0.7713	0.954	0.3511	0.945	0.3415	0.978

5. Real Life Application

Here, a real-world scenario is analyzed to showcase the applicability of the study. For this purpose, we consider the data set of successive failures time (in hours) of the air conditioning system of jet aeroplanes. The complete data set is given below:

Plane 7909: Y (n_2 *=29):* 90, 10, 60, 186, 61, 49, 14, 24, 56, 20, 79, 84, 44, 59, 29, 118, 25, 156, 310, 76, 26, 44, 23, 62, 130, 208, 70, 101, 208.

Plane 7913: X (n_1 *=27):* 97, 51, 11, 4, 141, 18, 142, 68, 77, 80, 1, 16, 106, 206, 82, 54, 31, 216, 46, 111, 39, 63, 18, 191, 18, 163, 24.

This data set is previously used by Agiwal [1] for the inverse Chen distribution. Before using the above data sets for our study first, we need to check that is these data sets fit the inverse Chen distribution or not. To check the fitting of the data sets, we use Kolmogorov-Smirnov (K-S) test. The value of K-S statistic and p-value for both the data set ***X*** are 0.2306 and 0.1132, respectively. Similarly, the value of K-S statistic and p-value for data set Y are 0.1438 and 0.5863, respectively. From the p-values obtained from the K-S test, we can say that both the data sets fit the ICD well.

The ML and Bayes estimates are now computed together with their respective ACI and HPD credible intervals. The Bayes estimates are calculated using $10,000$ MCMC samples with a burn-in period of $4,000$. The Bayes estimates are calculated in the case of non-informative prior as no prior information is available. To calculate the ML and the Bayes estimates of the parameters m is chosen such that $m = 35, 40, 45, 50$. The estimated values are shown in the following Tables 5 and 6.

Table 5. *The ML and Bayes estimates of* α_1, α_2 *and* θ *for the real data set.*

(n_1, n_2, m)	$\hat{\alpha}_1$		$\hat{\alpha}_2$		$\hat{\theta}$	
	MLE	Bayes	MLE	Bayes	MLE	Bayes
(27,29,35)	5.3470	4.7669	12.1899	9.9691	0.6011	0.5625
(27,29,40)	5.7256	5.2048	14.0294	11.5420	0.6420	0.6036
(27,29,45)	6.0172	5.5166	12.4561	10.7189	0.6448	0.6131
(27,29,50)	6.3544	5.8531	13.9942	11.9332	0.6776	0.6504

Table 6. The ACI and HPD credible intervals of α_1, α_2 and θ for the real data set.

(n_1, n_2, m)	$\hat{\alpha}_1$		$\hat{\alpha}_2$		$\hat{\theta}$	
	ACI	**HPD**	**ACI**	**HPD**	**ACI**	**HPD**
(27, 29, 35)	(2.6458,8.0481)	(3.1598,6.5246)	(3.3840,20.9959)	(6.3269,13.3289)	(0.4415,0.7608)	(0.4654,0.6624)
(27, 29, 40)	(2.9034,8.5477)	(3.2847,7.2269)	(4.0318,24.0271)	(7.4624,15.8579)	(0.4850,0.7989)	(0.5083,0.7097)
(27, 29, 45)	(3.1259,8.9085)	(3.5954,7.4526)	(4.1583,20.7540)	(6.9523,14.5628)	(0.4976,0.7920)	(0.5144,0.7164)
(27, 29, 50)	(3.3619,9.3469)	(3.8839,8.1174)	(4.7894,23.1989)	(7.7907,16.2899)	(0.5326,0.8225)	(0.5590,0.7709)

6. Conclusion

In this article, the problem of estimation of unknown parameters of inverse Chen distribution under joint type-II censoring was considered. To estimate the unknown parameters, the ML and Bayes estimates were obtained as point estimates with their corresponding interval estimates as asymptotic confidence and HPD credible interval estimates. The Bayes estimates were calculated under SELF. The Bayes estimates were obtained MCMC techniques. To compare the different estimates obtained throughout the study, a simulation study was carried out. From the results obtained in the simulation study, we can conclude that both ML and Bayes estimates provide good estimates of the unknown parameters. The Bayes estimates perform better than ML estimates in view of MSEs. The HPD credible intervals provide better interval estimates than ACI. All the interval estimates attain their nominal level of significance. Finally, for illustrative purposes, a pair of real data were analyzed.

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